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A COMPARISON OF TECHNIQUES FOR TIME DELAY ESTIMATION

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ABSTRACT

This paper reviews some of the techniques used for the identification of unknown lumped time delays in single input, single output (SISO) control systems and evaluates a number of these techniques in simulation and in implementation. Both off line and on line identification techniques are evaluated.

1. INTRODUCTION

A time delay may be defined as the time interval between the start of an event at one point in a system and its resulting action at another point in the system. Time delays are also known as transport lags, dead times or time lags; they arise in physical, chemical, biological and economic systems, as well as in the process of measurement and computation. Time delay estimation techniques may be broadly divided into off line techniques and on line techniques, with on line time delay estimation requiring continuous estimation of the time delay in a closed loop environment. Throughout this work, the author has largely concentrated on estimating the time delay (and often, simultaneously, the time constant) of a first order lag plus time delay (FOLPD) model of the process, as it is generally agreed that many processes may be represented by such a model for the purposes of acceptable PID controller tuning of the process.

1.1 Off line techniques

The off line time delay estimation techniques may be divided as follows:
(a) Methods using known process characteristics
(b) Analytical methods in the time domain
(c) Analytical methods in the frequency domain
(d) Graphical methods and
(e) Other methods.
The methods that use known process characteristics are largely "rule of thumb" based methods that are unsuitable for the estimation of time delays of unknown processes; even for known high order processes, the estimate of an equivalent time delay using these methods is not always adequate. The other four estimation techniques are evaluated in Section 2.

1.2 On line techniques

The on line time delay estimation techniques may be divided as follows:
(a) Methods that use rational approximations for the time delay in the continuous domain, followed by identification of the resulting process model parameters
(b) Methods that use sensitivity functions with a Smith predictor control scheme

(c) Overparameterisation of the discrete time process
(d) Statistical estimation methods and
(e) Other methods.

2. OFF LINE ESTIMATION METHODS INVESTIGATED

2.1 Analytical methods in the time domain

The method investigated was that described by Cheng and Hung [1], in which the parameters of the FOLPD model are estimated from the step response of the process, using least squares estimation. This method is referred to as the least squares (LS) method.

2.2 Analytical methods in the frequency domain

Three different techniques were investigated for the estimation of the time delay in the frequency domain:

(a) The measurement of two phase values at two frequency values and the calculation of the time delay (TD) and time constant (TC) of the FOLPD model using simultaneous equations; this method is referred to as the open loop phase/phase (OL P/P) method.
(b) The measurement of the gain and phase values at one frequency value and the calculation of the corresponding TD and TC values of the FOLPD model; this method is referred to as the open loop gain/phase (OL G/P) method.
(c) The measurement of the gain and phase values at one frequency value for a closed loop unity negative feedback system around the process; this method is referred to as the closed loop (CL) method. Details of this method are provided in the Appendix.

2.3 Graphical methods

The parameters of an FOLPD model are estimated graphically from the step response of the process. Three methods, attributed to Ziegler and Nichols [2], Miller [3] and Smith [4], respectively, are investigated; the values of TD and TC for each method are found as shown in Figure 1 below.

2.4 The use of a Limit Cycle to estimate the Time Delay

This method is a modification by the author of an idea outlined by Astrom and Hagglund [5] in which asymptotic tuning of a PID controller on a process may be achieved by analysis of the limit cycle provoked by the introduction of a relay in parallel with the controller. The author modified the method by introducing an integrator in series with the relay; the limit cycle is thus provoked when the phase lag of the process is 90^0. The amplitude and frequency of the limit cycle may be used, together with the off line closed loop method described in 2.2(c) above, to estimate the FOLPD model parameters.

3. ON LINE ESTIMATION METHODS INVESTIGATED

3.1 Overparameterisation of the discrete time process

Time delay may be estimated by the technique of overparameterisation of the discrete time process, followed by the recursive identification of the model parameters with empirical rules to estimate the time delay. This method is based on the papers by De Keyser [6] and Wong and Bayoumi [7].

The process with a time delay is described by the difference equation:

$$y(t) + a_1 \times y(t-1) + ... + a_{n_0} \times y(t-n_0) = b_0 \times u(t-d_0) + ... + b_{m_0} \times u(t-d_0-m_0) + v(t) \quad (1)$$

where v(t) = noise function and d_0 = unknown time delay index of the process (time delay index = time delay/sample period).
The model for the process is described by the difference equation:

$$y(t) + A_1 \times y(t-1) + ... + A_n \times y(t-n) = B_0 \times u(t-d) + ... + B_m \times u(t-d-m) + e(t) \quad (2)$$

where d = estimate of the time delay index (chosen such that it is less than or equal to d_0) and m and n are chosen such that m and n are greater than or equal to $m_0 + d_0 - d$ and n_0 respectively; e(t) is the model error. Then, in theory,

$$B_i = b_{i-d_0+d} \; , \; d_0-d \le i \le m_0+d_0-d \quad (3)$$

and $B_i = 0$ otherwise.
The recursive least squares algorithm is used to estimate the parameters A_i and B_i; the correct time delay index value is estimated as d plus the index i of the first $|B_i|$ value to lie above a defined threshold level. This method (from Wong and Bayoumi [7]) is used to estimate the initial value of a time varying time delay index. De Keyser [6] postulates that this initial value may be improved by changing it according to the results of two inequality equations which use the identified model parameters as data i.e.

if $|B_0| > 5*|B_m|$ .... decrease time delay index and
if $|B_0| < |B_m|/5$ .... increase time delay index,

where B_0 and B_m are as given in equation (2).
These equations come from examination of the variances of sequences obtained by subtracting the outputs of overparameterised models based on successive time delay indices, when the input to the models is a white noise signal.

The author expands on the work by De Keyser to allow fractional time delays to be estimated. If the transfer function in the s domain is modelled as a FOLPD model, identifying the parameters of the z domain equivalent of this transfer function will directly give the fractional part of the time delay. The transfer function of a FOLPD model in the s domain is

$$G(s) = \frac{K \times e^{(-s \times TD)}}{1+s \times TC} \quad (4)$$

The digital equivalent of G(s) is

$$(1-z^{-1}) \times Z[\frac{K \times e^{(-s \times TD)}}{s \times (1+s \times TC)}] \quad (5)$$

with TD = g.T - h.T, g = integer, h = fraction and T = sample time.
It is straightforward to show that G(z) equals

$$\frac{[K\times(1-e^{\frac{-h\times T}{TC}})]\times z^{-(g+1)}+[K\times(e^{\frac{-h\times T}{TC}}-e^{-\frac{T}{TC}})]\times z^{-(g+2)}}{1-(e^{-\frac{T}{TC}})\times z^{-1}} \tag{6}$$

Identification of the terms of G(z) will allow h to be calculated.

3.2 Modelling Time Delay by a Rational Approximation

A method proposed for time delay estimation is to model it by a rational approximation, use recursive identification to find the resulting system parameters and from these parameters calculate the time delay. The method evaluated is based on modelling the time delay by a zero in the s domain, as proposed by Roy, Malik and Hope [8]; the parameters of the model are then identified recursively in the z domain.

4. SIMULATIONS AND EXPERIMENTAL RESULTS

The methods of time delay estimation were investigated in simulation and/or in implementation. The processes used for implementation of the methods were the PCS327 process control simulator, in which a number of processes from a first order lag to a third order lag plus time delay may be constructed, and the PT326 process trainer, which is a system in which air is drawn by a centrifugal blower past a heated grid and through a length of tubing back to the atmosphere. Both of these systems are supplied by Feedback Instruments Ltd., Crowborough, Sussex, England. The step responses of the processes in open loop or closed loop, as appropriate, are obtained using LPCLAB, a DT-2811 data acquisition board and a PC77 data interface board with a PC; LPCLAB is a real time software product, written in assembly code, that facilitates A/D and D/A transfers using the DT-2811 board.

Figure 2 below shows the step response obtained from a second order lag plus time delay set up on the process control simulator, together with some model step responses. As mentioned in Section 2, "LS model" refers to the model derived using the method of Cheng and Hung and "CL freq model", "OL G/P model" and "OL P/P model" refer to the models found using the three analytical methods in the frequency domain. The "RSS error" is a measure of the goodness of fit of the model to the process; it is the square root of the sum of the squares of the error between the process step response and the model step response. Figure 3 shows similar plots to Figure 2, with the model step responses found using graphical methods. Figures 4 and 5 show the step responses obtained from the process trainer, together with the corresponding model step responses.

The plots show that, broadly speaking, the parameters identified using the methods above give models that are a good approximation to the original process, with the Ziegler-Nichols method providing the least satisfactory parameters. Independent plots of the estimates of TD versus model type show that the estimates of time delay given by all methods for a particular process are broadly similar. The estimates of TD and TC obtained are dependent on the open loop DC gain value taken; since this is measured experimentally for the frequency response methods and is evaluated from the actual step response data in the graphical and LS methods, the latter methods should have a lower error value than the frequency response methods. More accurate measurement of the open loop DC gain would appear to indicate a lowering of the error value for the frequency response methods, which would suggest that the CL frequency method, in particular, may be as good as the LS method for FOLPD model parameter estimation.

Figures 6 and 7 below show the step responses obtained from a second order process plus time delay, and from the process trainer, together with the step responses found using the LS model and the CL relay method (Section 2.4 above), with d being the magnitude of the relay. The CL relay method gives the best model when d = 2; sources of error in estimating the model parameters using this method are inaccurate measurement of the open loop DC gain, and inaccurate measurement of the amplitude of the limit cycle. It can be shown that the latter inaccuracy has a disproportionate effect on the TD and TC values identified. Overall, the parameter values identified (when d = 2) appear sufficiently accurate for adequate tuning of a PID controller for the system.

The method of overparameterisation as a method of estimating the time delay is evaluated in simulation; as mentioned in Section 3.1, both multiple and fractional time delays may be estimated by this method. Figures 8 and 9 show the delay index estimate tracking the actual value of the delay index as it is varied.

The method works well for FOLPD processes, though it will not estimate the equivalent time delay of a process without an explicit time delay term, as such a process in incapable of being modelled in the required form in the z domain. The method appears best suited to estimate the time delay of systems with small time delays (so that the total number of parameters to be estimated is not too large), with a time delay value that does not vary rapidly or by great amounts.

The modelling of the time delay by a zero in the s domain, translating the model to the z domain and then identifying the resulting system to attempt to estimate the time delay, did not work for simulations taken (i.e. it was not possible to estimate the time delay from the resulting system parameters identified). Thus, underparameterisation of a time delayed process by a model will not allow the estimation of the time delay.

Fig. 1 : Some graphical methods of estimating FOLPD model parameters

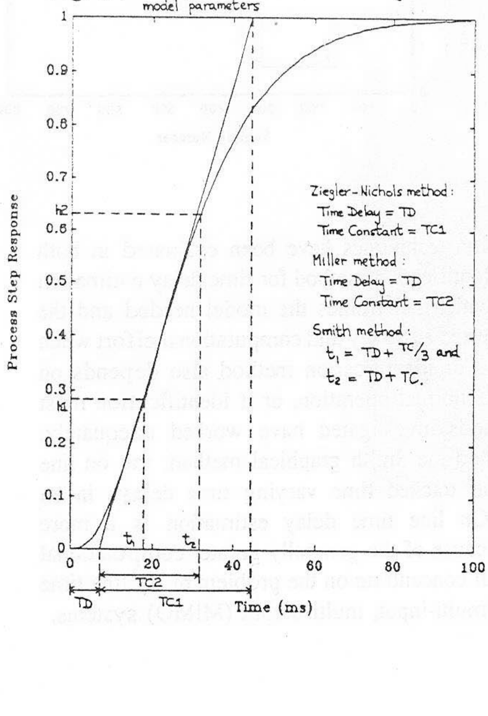


Fig. 2 : Process Simulator - Second Order Lag plus Delay - Process,Models

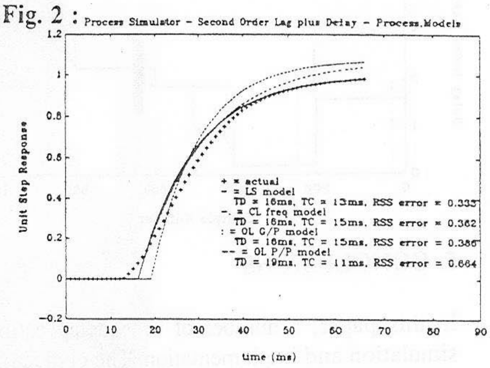


Fig. 3 : Process Simulator - Second Order Lag plus Delay - Process,Models

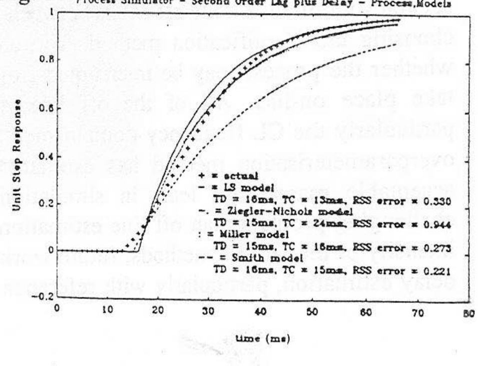


Fig. 4 :

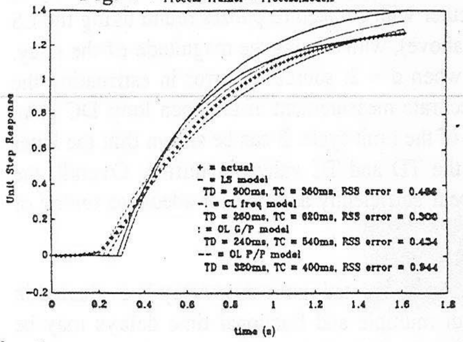


Fig. 5 :

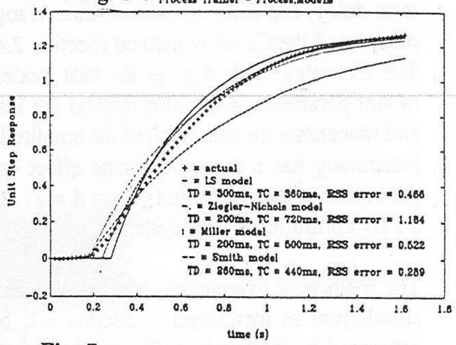


Fig. 6 :

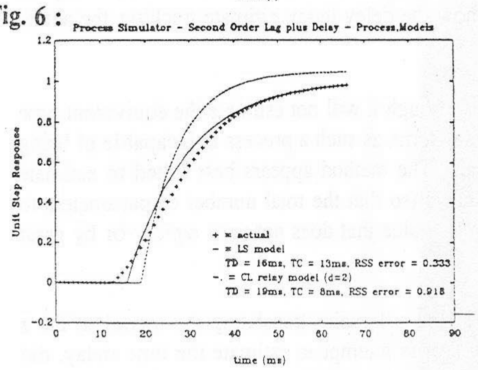


Fig. 7 :

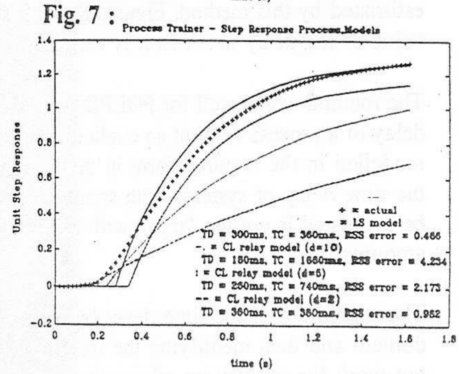


Fig. 8 :

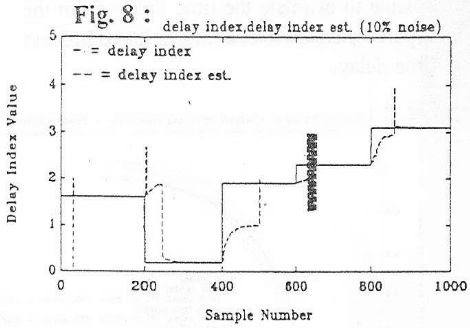
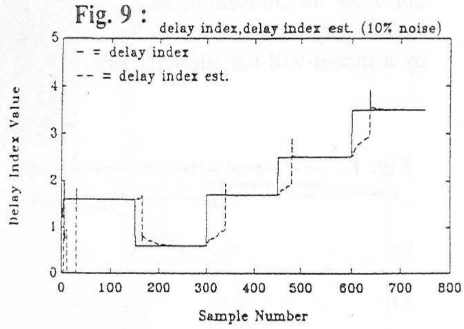


Fig. 9 :



5. CONCLUSIONS

In this paper, a number of time delay estimation techniques have been evaluated in both simulation and implementation. The choice of identification method for time delay estimation depends on the purpose of the identification, which determines the model needed and the accuracy required; a trade off exists between required accuracy and computational effort when choosing the identification method. The choice of identification method also depends on whether the process may be interrupted from its normal operation, or if identification must take place on-line. All of the off line methods investigated have worked adequately, particularly the CL frequency domain method and the Smith graphical method; the on line overparameterisation method has estimated and tracked time varying time delays in an acceptable manner (at least in simulation). On line time delay estimation is a more challenging problem than off line estimation because of the generally greater computational intensity of the on line methods; future work will concentrate on the problem of on line time delay estimation, particularly with reference to multi-input, multi-output (MIMO) systems.

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APPENDIX : Estimating TD and TC of an FOLPD model using the CL method

The unity feedback closed loop transfer function around a FOLPD model is

$$\frac{\dfrac{K\times e^{(-s\times TD)}}{(1 + s\times TC)}}{1 + \dfrac{K\times e^{(-s\times TD)}}{(1 + s\times TC)}} = \frac{K\times e^{(-s\times TD)}}{1 + s\times TC + K\times e^{(-s\times TD)}} \qquad (A1)$$

The equivalent frequency transfer function is

$$\frac{K\times[\cos(w\times TD) - j\times\sin(w\times TD)]}{1 + j\times w\times TC + K\times[\cos(w\times TD) - j\times\sin(w\times TD)]} \qquad (A2)$$

Expressing this in polar form gives

$$K1 = \frac{K}{\sqrt{[1 + K\times\cos(w\times TD)]^2 + [w\times TC - K\times\sin(w\times TD)]^2}} \qquad (A3)$$

and

$$\phi = -w \times TD - \tan^{-1}\left[\frac{w \times TC - K \times \sin(w \times TD)}{1 + K \times \cos(w \times TD)}\right] \tag{A4}$$

Equation (A4) implies that

$$-\tan(\phi) = \tan\left[(w \times TD) + \tan^{-1}\left(\frac{w \times TC - K \times \sin(w \times TD)}{1 + K \times \cos(w \times TD)}\right)\right] \tag{A5}$$

$$= \frac{\tan(w \times TD) + \dfrac{w \times TC - K \times \sin(w \times TD)}{1 + K \times \cos(w \times TD)}}{1 - [\tan(w \times TD)] \times \left[\dfrac{w \times TC - K \times \sin(w \times TD)}{1 + K \times \cos(w \times TD)}\right]} \tag{A6}$$

i.e.

$$-\tan(\phi) - \tan(w \times TD) = \left[\frac{w \times TC - K \times \sin(w \times TD)}{1 + K \times \cos(w \times TD)}\right] \times (1 - [\tan(\phi)] \times [\tan(w \times TD)]) \tag{A7}$$

i.e.

$$\frac{w \times TC - K \times \sin(w \times TD)}{1 + K \times \cos(w \times TD)} = -\left[\frac{\tan(\phi) + \tan(w \times TD)}{1 - [\tan(\phi)] \times [\tan(w \times TD)]}\right] \tag{A8}$$

$$= -\tan(\phi + w \times TD) \tag{A9}$$

i.e.

$$TC = \frac{K \times \sin(w \times TD) - [1 + K \times \cos(w \times TD)] \times \tan(\phi + w \times TD)}{w} \tag{A10}$$

Equation (A3) implies that

$$[1 + K \times \cos(w \times TD)]^2 + [w \times TC - K \times \sin(w \times TD)]^2 = \left(\frac{K}{K1}\right)^2 \tag{A11}$$

Substituting equation (A10) into equation (A11) gives

$$[1 + K \times \cos(w \times TD)]^2 + [1 + K \times \cos(w \times TD)]^2 \times \tan^2(\phi + w \times TD) = \left(\frac{K}{K1}\right)^2 \tag{A12}$$

i.e.

$$[1 + K \times \cos(w \times TD)]^2 \times [1 + \tan^2(\phi + w \times TD)] = \left(\frac{K}{K1}\right)^2 \tag{A13}$$

The value of time delay may be calculated from this identity.